A Model of Devaluation-Inflation Cycle Phenomena

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Abstract
Using a model of dual foreign exchange markets (an official market with a crawling-peg foreign exchange rate and illegal parallel market with a free-floating rate), the paper shows that the adoption of a restrictive import policy in the official market raise the equilibrium parallel market premium rather lower it. The paper also shows that under a fixed (or adjustable peg) exchange rate policy and, with an increasing domestic money supply the use of periodic devaluations of the official exchange rate, in order to restore official reserve loss, could lead to a devaluation-inflation spiral and to eventual collapse of the fixed exchange rate regime.

Introduction

The increasing interest in studying parallel markets for foreign exchange in developing countries over the past few years is possibly due to the increasing size of these markets despite successive attempts by some governments in the past decade to reduce the impact of parallel markets on official reserves(1). It has become apparent to many economists in recent years that a sound foreign exchange policy should consider the link between the parallel market for foreign exchange and the rest of the economy.

To contain the rising parallel market premium (ratio of a parallel market rate to an official rate) and restore official reserve loss (due to rising premium) authorities in some countries have resorted instead to more restrictive foreign exchange policies in the official market by restricting import in the official market and in some cases by adopting periodic devaluations in an attempt to anchor the parallel market rate. This paper shows when the growth in the domestic money supply is out of control (due to permanent fiscal deficit) (2) and domestic reserve are insufficient to support official demand, devaluing the official exchange rate to restore official reserve loss could lead to a devaluation-inflation-spiral and to eventual collapse of the fixed exchange rate system [3].

The paper has been organised as follows. The first part of the paper sets up macroeconomic model with a parallel market for foreign exchange. The second part includes steady state analysis under a crawling-peg foreign exchange system, in particular to assess the impact of import controls on the equilibrium parallel market premium. The final part of the paper shows a devaluation-inflation spiral that leads to the collapse of a fixed exchange rate regime.

2. The Macroeconomic Model

The macroeconomic model is made up of the optimal rules of the various decision-making agents in the economy. Of these agents, private sector producers choose output and input levels for both home goods and export goods. Inputs are of two types; labour and imported. Firms also set how much of any export revenue to divert to the foreign exchange market by under-invoicing export sales. Households choose how much of their total financial wealth to hold in domestic currency, and how much to hold in foreign currency. Finally, the government determines its fiscal stance and financing, and the rules for pricing and rationing in the official foreign exchange market. We will deal with each in turn, and then assemble the various decision rules into equilibrium conditions for the two parts of foreign exchange market.

i. Domestic producers' decisions

Domestic firms can produce for home consumption or export. Exports are exogenously determined, as they depend mainly on international terms of trade. Home goods (y) have a Cobb-Douglas production function with imported producer goods I_p and domestic L_y as the only inputs:

$$(1)\quad Y \le I_p^{(1-\alpha)} L_y^{\alpha}, \qquad 0 \le \alpha \le 1$$

The domestic currency purchase price of imported producer goods is the foreign currency price P_m multiplied by the exchange rate relevant for competitive purchases. Competitive purchases of imported producer goods will always be made at the parallel market exchange rate (that is, price of foreign currency) b, since the government imposes import restrictions for purchases at the official exchange rate(4).

Firms have a choice about converting the foreign currency revenue from their exports P_x X (where P_x is the foreign price of exports, for convenience we set $P_x = 1$) into domestic currency: they can do so in the official market at the official exchange rate e (which is exogenous) or in the parallel market at the higher rat b. They can divert a portion Φ of their foreign currency revenue from the official market to the parallel market, despite foreign exchange controls, by under-invoicing. The size of the chosen amount will reflect both the expected gain from conversion at the higher parallel rate b, and the probability of being detected and penalised (losing the entire transaction ($\Phi\ bX$) through confiscation). We assume that the probability of detection (or risk of penalty) rises with the proportion of under-invoicing, and for simplicity at exactly half the rate, i.e. the probability is $\Phi/2$, and the expected confiscation amount is therefore $(\Phi/2)\ (\Phi bX)$. Firms' decision rules for all the choices above are found by maximising their profit function.

$$(2)\ MAX\ [P_yY + \{\Phi b\ (1 - \Phi/2) + (1 - \Phi)\ e\}\ X - bP_m\ I_p - W\ (L_x + L_y)]$$

With respect to L_y, I_p and Φ, subject to production technology constraint and to the usual non-negativity restrictions. The first order conditions for I_p, L_y and Φ are:

$$(3)\ P_y\ (1 - \alpha)\ I_p^{-\alpha}\ L_y^{\alpha} = P_m b$$

$$(4)\ P_y\ \alpha Y / L_y = W$$

$$(5)\ \Phi = (1 - 1/\pi)$$

Where $\pi = b/e$, refers to the parallel market premium. Concavity of equation (5) implies that, while rising premium induce under-invoicing of export revenue, it also raise the risk of penalty due to higher detection probability.

goods is a constant multiple of the domestic currency price of imports:

With constant average labour productivity of home goods y/L_y it can be shown using equation (3) and (4) that the price of home

(6) $P_y = \sigma Pm\ b$,

where $\sigma = [1/\alpha^{\alpha}) (1 - \alpha)^{\alpha-1} (\alpha Y/ L_y)^{\alpha}]^{(1/(1-\alpha)}$

Then from (6) domestic inflation can be expressed as:

(7) $\hat{P}_y = \hat{P}_m + \hat{b}$

Where a hat over a variable denotes a proportional change, $\hat{X} = \mathring{X}/X$

Equation (3) can be rearranged to solve for imports of producer goods. Substituting for P_y from equation (6) as well, the optimal level of imported producer goods is a constant multiple of output Y:

(8) $I_p = \sigma (1 - \alpha)Y$

Where I_p is imported producer goods.

***ii.* Household portfolio allocation:**

Households choose between domestic and foreign assets, a portfolio allocation decision. Households' nominal financial asset portfolio H is assumed to consist only domestic money holdings M, and foreign money holdings F. Since households buy foreign currency F only in the parallel market, and therefore value it at the parallel market exchange rate b, the domestic currency value of households' nominal wealth H can be expressed as:

(9) $H = M + bF$

Let λ be the fraction of financial wealth H that households want to hold in foreign currency. Both foreign currency F and domestic currency M earn zero interest, but F will provide a return whenever the parallel market rate b changes[(5)]. The fraction λ will therefore rise with the actual rate of increase in the parallel market rate[(6)].

In equilibrium, desired holdings of foreign money λH must equal the actual stock bF of foreign money being held, so we can solve for $H = bF/\lambda$. Replacing H in equation (9) and rearranging to solve for M, and dividing both sides by, e, to convert to (official) foreign currency values giving:

(10) $m = [(1 - \lambda)/ \lambda)]\ \pi F$

where m = M/ e. The fraction λ is a function of the rate of increase in parallel market rate, but this can be broken down into appreciation of the official exchange rate, $\hat{e}$, and of the parallel market premium π. Letting $\Lambda(\pi + \hat{e})$ stand for the relationship of $(1 - \lambda)/\lambda$, we can rewrite equation (10) as:

(11) $m = \Lambda(\pi + \hat{e})\,\pi F \quad \Lambda' < 0$

Equation (11) is the portfolio-balance or the asset market equilibrium condition. It indicates that the higher the expected rate of increase of the parallel market rate (that is, depreciation of domestic currency in the parallel market), the lower is the ratio of domestic money holdings to foreign currency holdings.

iii. Government decisions

The government determines much of the context for decisions of other agents in the economy, and also acts as a separate agent. For instance, the government decrees and administers both an import quota system and a set of foreign exchange controls which regulate entry into the official exchange market. In this market the government buys foreign currencies from households at the official exchange rate e, and allocates it either to pay for government imports (G) or to sell to households for officially sanctioned imports. The government can buy from only one source: private sector export revenue X.

We assume that government spending G is entirely on imports, including payment of interest on foreign debt, and that no new foreign debt is being incurred. Further, we assume that any of G that is not financed by taxes must be financed by borrowing from the Central Bank. Since G is all spent on imports, we assume that G is fixed in foreign currency units. That is, $G/e = g$ is constant.

The change in the stock of domestic money M, is equal to the change in central bank domestic credit, D, plus the change in (domestic currency value of) foreign reserves held by the government, eR. The change in domestic credit reflects government borrowing from the central bank to finance its deficit, $G - T$ or (in foreign currency) $g - t$ (where t=T/e) that is,

(12) $\mathring{m} = (g - t) - m\,\hat{e} + \dot{R}$

Since $M/e = m + m\hat{e}$.

The value of R is determined in the official foreign exchange market, to which we now turn.

***iv.* Foreign exchange markets**

(a) The official market:

The official current account balance[7], R, is determined as a fraction of export revenue channeled through the official exchange market [$(1\text{-}\Phi)\ eX$] less government imports (G) and private sector purchases of official imports, all valued in foreign currency. In order not to leave the official import undetermined, we assume that the government sets a fixed proportion δ of the total foreign currency inflow to the official market for private imports. Algebraically, the official current account balance can be expressed as:

(13) $R = (1 - \delta)\ [(1 - \Phi)\ X] - g.$

Substituting for Φ (equal to ($1\text{-}1/\pi$)) from equation (5) yield the final form of the official current balance:

(14) $R = ((1 - \delta)/\ \pi)\ X - g.$

(b) The parallel market:

In the parallel foreign exchange market, the supply of foreign currency consists of the portion Φ of export revenue X that is diverted from the official market by under-invoicing. Demand consists of the current account flow of imports not permitted under the official import policy, and increases in households' stock of foreign currency. As a result, the current account balance in the parallel market, F, is determined by subtracting total imports (official and unofficial, private sector and government), capital flight [8], and accumulation of official reserves, from the total inflow of foreign currency to the economy:

(15) $F = X - (I_p + g) - C - R.$

where I_p is imports of producer goods, and C is the capital flight measured in foreign currency units. We assume the amount of capital flight is a fixed proportion, a, of total private financial wealth, (m + πF). Substituting for R from equation (14) and I_p from equation (8) we get the final form of the current account balance in the parallel market:

(16) $\mathring{F} = [1 - (1 - \delta)/\ \pi)]\ X - \sigma\ (1 - \alpha)\ Y - a\ (m + \pi F).$

Equation (16) completes the model, which consist of the differential equation system (11), (12), (14) and (16) in m, F, R and π. To simplify, we reduce the differential equation system to three equations by substituting for R into equation (12) giving

(17) $\mathring{m} = \quad X - m\hat{e} - t$

The third differential equation is (11), repeated here:

(11) $\mathring{m} = \Lambda\ (\hat{e} + \pi)\ \pi F \qquad \acute{\Lambda} < 0$

In the next section we establish the steady state values of the system represented by equations (11), (16), and (17).

3. Steady State and Stability

The steady-state values of π, F, and m, when the rate of change of the official rate is exogenously determined constant ($\hat{e} > 0$), are determined by solving for π, F, and m, when $\pi = 0$, $F = 0$ and $m = 0$. To solve for the steady state values we set $F = 0$ in equation (16) and substitute from equation (11) for F to get:

(18) $\alpha (1 + 1/\Lambda)\, m = [1 - (1 - \delta)/\pi]\, X - I_p$

Setting $m = 0$ in equation (17) obtains:

(19) $((1 - \delta)/\pi)\, X = m\hat{e} + t$

Solving for m from equations (18) and (19) and denoting $[X - (I_p + t)] = \eta$ we get

$$(20)\ \bar{m} = \frac{\eta\Lambda}{[A\,(1 + \Lambda) + \hat{e}\Lambda]}$$

Substituting (20) into (19) and solving for π:

$$(21)\ \bar{\pi} = \frac{(1 - \delta)\, X\, [a\,(1 + \Lambda) + \hat{e}\Lambda)]}{}$$

Finally from (11), (20) and (21) we get:

$$(22)\ \bar{F} = \frac{\eta\, [\hat{e}\Lambda\,(\eta + t) + a\,(1 + \Lambda)\, t}{}$$

Since $b = \hat{e} + \hat{\pi}$, then in the steady state ($\pi = 0$), Λ, can be expressed as function of $\hat{e}$ only.

Equation (21) shows that the parallel market premium rises if the government allocates a smaller share δ of total official foreign currency receipts to private sector import quotas. That is, tightening of import quotas on private sector imports, as practiced by many developing countries facing a shortage of foreign currency, will by itself lead to a higher premium level in the steady state as seekers of foreign currency move from the official to the parallel market.

In order to analyse the composition of private sector portfolio in the steady state we use equations (20), (21) and (22) to get:

(23) $m/F = \bar{\pi}\, \Lambda\, (\hat{e})$

Since $\acute{A}(\hat{e}) < 0$, equation (23) implies that any increase in the official exchange rate depreciation will lower the ratio of domestic currency to foreign currency held in private sector portfolio. This indicates that in the steady state, depreciation of the official exchange rate increase demand for foreign currency in the parallel market. As a result we expect a positive correlation between change in the official exchange rate and change in the parallel market rate. This relationship between the official rate and the parallel rate will be utilized in the coming section.

Establishing the stability of the model and showing existence of a unique solution for the steady state values is more difficult (9). Analysis of the characteristic polynomial of the linear approximation of the equations (11), (16), and (17), shows that the dynamic model has one positive and two negative roots. That is, the steady state discussed above is a saddle-point solution, therefore the economy can (re-) converge to the steady state from a distance away.

The following section investigates devaluation-inflation spiral and the collapse of fixed exchange rate regime.

4. Devaluation-inflation Spiral

This section shows that under a fixed (adjustable peg) official rate policy and increasing rate of domestic money supply (due to expanding budget deficit caused by rigid official import policy), official rate devaluation as an instrument to replenish a loss in official reserves could lead to a devaluation-inflation-spiral, and to eventual collapse of the fixed foreign exchange policy.

In order to model the devaluation-inflation a few adjustments have to be introduced to some of the equations of the model. Let us consider a small open economy, in which the foreign prices of traded goods (export and import prices) are taken as given (for simplicity equal to 1), and production of export goods decreases with the appreciation of real official rate (the nominal official rate divided by the domestic price level). It is also assumed that a devaluation of the nominal official exchange rate takes place when the official reserves reach a critical minimum level which correspond to a premium level denoted π_H, otherwise the official rate is fixed. Given that the parallel market rate always adjust to maintain equilibrium in private current account, the model can be expressed as:

(25) $\Delta e = \beta\ (\pi - 1)$

$\beta > 0, \text{ if } \pi = \pi_H$,

$\beta = 0 \quad \text{if } \pi < \pi_H$

(26) $m = \Lambda\ (\pi + \hat{\beta}\ (\pi - 1))\ \pi F$

(27) $\mathring{m} = (g - t) + \dot{R}$

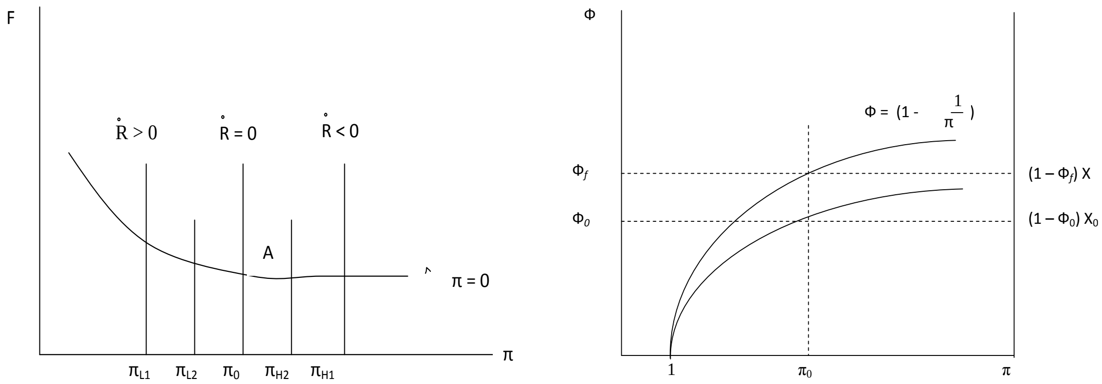
where Δ stand for a change.

Suppose at the initial state the deficit, and the official current account are balanced so that,

(28) $\mathring{m} = \mathring{R} = (g - t) = 0$

The premium level that maintains official current account balance in the initial state is denoted by π_0, Now, let us introduce a government budget deficit by reducing the real tax revenue below the real government spending (which is constant). The resulting increase in money supply raises the parallel market rate and therefore generates a disequilibrium condition. The dynamics of the devaluation-inflation spiral resulting from such a disequilibrium condition is shown in the figure below, which uses equations (25), (26), and (27).

Fig (c): Devaluation-Inflation Spiral



Before the fiscal deficit is introduced the economy is at a steady state equilibrium at point A, in the upper diagram, for which the growth in real domestic money is Stationary ($m = 0$), the portfolio assets are at equilibrium ($\hat{\pi} = 0$) the official and the private current accounts are balanced ($\dot{R} = 0$, $\dot{F} = 0$). In the lower diagram, export under-invoice, Φ, and the official export revenue, $(1 - \Phi) X$, are expressed as functions of the parallel premium. Each curve of export under-invoice corresponds to a given level of export production. As export production decreases export under-invoice curve shift upward to ensure a higher ratio of under-invoice that maintains balance in private current account.

With the introduction of a fiscal deficit ($(g - t) > 0$), the resulting growth in domestic money supply introduces a higher premium level that raise the premium towards (π_{H1}), as the excess of domestic money raise the demand for foreign currency in the parallel market. The rising premium will increase under-invoice of export and the official reserves will run down till they reach the critical level at π_{H1}; at which the government devalues the official rate so that a premium level that reduces the under-invoice of export is restored[(10)]. At the premium level π_{L1} official reserves loss is restored temporarily, but the official exchange rate devaluation reduces further the real tax revenue ($t = T/e$), and expands the fiscal deficit. The expansion in the fiscal deficit ($g - t$), and the resulting acceleration in domestic money growth, raise the depreciation of the parallel rate above the official rate devaluation[(11)].

As a result, each devaluation event generates an appreciation of the real official exchange rate, which reduces the production of export goods. Since after each devaluation episode the government and private sector compete for a smaller amount of export revenue then for a given government imports (g), the critical level of the official reserves is reached at a lower premium level. Therefore, in the upper diagram when the official reserves reach its critical level at π_{H2}, a devaluation of the official rate follows as before. The government again restores its official reserves' loss at, π_{L2} and the subsequent appreciation in the real official rate generates further reduction in total export production. In general, the increasing growth in domestic money and the subsequent appreciations of real official exchange rate generate more frequent devaluations that eventually bring the critical official reserve towards the parallel premium π_0, but at a lower export production which corresponds to the highest curve in the lower diagram. In order to show that π_0 is no longer the premium level that maintains official reserve balance it is important to realize that the official export revenue that maintains official reserves balance is $(1 - \Phi_0) X_0$ (in the lower diagram). But after the series of devaluations and subsequent appreciations of the real official exchange rates, the official export revenue reduced to $(1 - \Phi_f) X_f$. The amount of export revenue, $(1 - \Phi f) Xf$ is less than the amount of export revenue that maintains official reserve balance. Since further devaluation of the official rate will not improve the situation, but on the contrary, it will make it worse, the only remaining option is to abandon the fixed exchange rate policy and unify both rates at $\pi = 1$, as shown in the lower diagram. This shows the eventual constraint the government will be facing when its policy is to maintain a fixed flow of government imports while maintaining an official exchange rate that only respond to an official reserve loss.

5. Conclusion

The primary objective of this paper is to show when there is an active parallel market for foreign currency in the economy, and rising growth in domestic money supply (caused by expanding fiscal deficit), the adoption of periodic devaluations of the official exchange rate (the often used approach in many developing countries facing deteriorating official reserves) could generate devaluation-inflation spiral that leads to eventual collapse of the fixed exchange rate system.The devaluation-inflation spiral invoked simply because of volatility of official reserves on one hand, and rigidity in government imports on the other hand. Such a situation necessitates periodic devaluations that fuel domestic inflation, and that adversly affect export revenue (through its effect on real exchange rate). The decline in total export revenue contributes to official reserve loss, and as it reaches a critical minimum level further devaluation invoked and a new cycle of inflation is initiated. A policy implication of this study is that a policy of official exchange rate that only respond to official reserve loss is not sustainable when government imports are perfectly inelastic and official reserves are volatile.

Endnotes:

(1) For an excellent review article on different models of parallel market for foreign exchange and their empirical findings see Kiguel and O'Connel (1995).

(2) The permanent fiscal deficit is caused by rigid official import policy that fails to match demand for foreign currency in the official market with the available resources of foreign currency in the official market. The rigidity of the official imports is due to perfectly inelastic nature of government imports (i.e. equipments for military and other public goods.).

(3) Rodriguez (1978) modeled the devaluation-inflation spiral in a single foreign exchange market model with the objective of detecting the causal relationship between inflation and devaluation. In this paper using a model of parallel market for foreign exchange, I have shown that the devaluation- inflation spiral is a phenomenon that precedes the collapse of a fixed exchange regime.

(4) Lizondo (1987) also uses this assumption and he conclude that, under a reserve rationing regime those who obtains foreign exchange at the official exchange rate receive a rent that they can collect by selling the imported goods at the parallel exchange rate. Prices of all imported goods are also likely to reflect the parallel market rate when foreign currency obtained at the official market leaks into the parallel market. The presence of cross-market transactions under dual foreign

exchange regimes in some developing countries has been realized and investigated by Jagdeep and Vegh (1990).

(5) A principal motive of holding domestic money despite its zero interest earning, because of zero risk of holding it.

(6) Lizondo (1987), and Dornbush (1983) use perfect foresight assumption as they assume that λ rise with the expected increase of the parallel rate.

(7) There are no capital account transactions in the official market.

(8) Capital flight in this context is defined as the illegal outflow of foreign currencies for reasons other than imports.

(9) A prove of the stability condition is available from the author upon request .

(10) Official exchange rate devaluation reduces the premium level only if $db/de < 1$, however, this assumption is not necessary for the collapse of fixed exchange rate regime since a government cannot run permanent balance of payment deficit while maintaining a fixed exchange regime .

(11) This is because the parallel market rate change is reflecting the official exchange rate change (as shown by equation 23) and also domestic money growth rate.

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